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A quark model estimation of quark-pion coupling and nucleon-pion coupling constants

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Abstract : Incorporating chiral symmetry to the potential model of quarks with confining potential $V_q(r) = (1+\gamma^0)\, a \ln(r/b)$, $a, b > 0$ that gives a reasonable quark-core contribution to the static nucleon properties, the quark-pion (qqπ) coupling and nucleon-pion (NNπ) coupling constants for quarks in a nucleon are estimated. The values of $(G^2_{qq\pi}/4\pi)$ obtained with the approximation of a point pion and finite size pion are quite comparable with those extracted from the experimental vector-meson decay-width ratios by Suzuki and Bhaduri. The NNπ-coupling constant $(g^2_{NN\pi}/4\pi)$ comes out to be 14.539 in reasonable agreement with the experimental value 14.1.

Keywords : Quarks, potential model, chiral symmetry, quark-pion and nucleon-pion coupling constant

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1. Introduction

The static electromagnetic properties of low lying baryons can be explained reasonably well [1] in a phenomenological model where one considers the quarks as point Dirac particles moving independently in an effective potential taken as an equal admixture of scalar and vector parts. However, unlike the electromagnetic and isospin currents, the axial vector current carried by the quarks is not conserved in this model. Such a situation is inherent with all the potential models confining quarks including the bag model. But in view of the experimental success of PCAC and hence the fact that chiral SU(2) × SU(2) is one of the best symmetries of strong interactions, it is desirable to conserve the total axial vector current in such models describing hadrons. This is usually done at a

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phenomenological level in chiral bag models [2], by introducing elementary pion field that also carries an axial current such that the four divergence of the total axial vector current satisfies the PCAC condition. In spite of many successful applications of chiral bag models, it is not totally free from certain objections; particularly, for its insistence on excluding pions from the interior of static spherical bag. Therefore, there have been in the recent past, several attempts to formulate alternative schemes with confining potentials of appropriate Lorentz structures. The chiral potential models [3] are no doubt some attempts in this direction. Effective potentials having Lorentz structures with equally mixed scalar-vector parts in linear [4], harmonic [5], Power-law [6] and logarithmic [7] form have also been investigated in this context. All these potential models, used in a limited baryonic sector, only meet with a success more or less identical to the bag model. However, this does not establish the non-uniqueness of bag-like models unless such potential models are pursued extensively over a much wider range of hadronic phenomena. The present work is only a step in that direction where the chiral potential model with an equally mixed scalar and vector logarithmic potential model [7] in the form

$$V_q(r) = (1+\gamma^0)[a\log(r/b)], \quad a, b>0 \tag{1.1}$$

which was used earlier in explaining the static electromagnetic properties of baryons, is extended to investigate the coupling of quarks to pions in a nucleon. In this work, we are mainly interested to estimate the quark-pion (qqπ) coupling and nucleon-pion (NNπ) coupling constants in a nucleon with the model (1.1).

We follow here the usual procedure to incorporate chiral symmetry in the framework of this potential model. As a consequence of preserving the chiral symmetry in the model, there appears a possible residual interaction of the core-quarks with the surrounding elementary pions. This residual interaction must provide additional contributions to the baryonic properties over and above the core contributions. However, since the core contributions are already found [1] to be quite close to the experimental values, the extra pionic contributions must be small. This, therefore, suggests that the residual quark-pion interaction is perhaps quite weak to be treated in low-order perturbation theory. The quark-pion interaction Lagrangian density $\mathcal{L}_I^\pi$ in this model may be found to be proportional to the scalar part of the confining potential which in a way decides the strength of the quark-pion coupling. Therefore, our main objective here is to determine the quark-pion coupling constant in this model to examine its consistency with the estimates made earlier by other workers [8].

2. Chiral symmetric potential model

The quarks in a hadronic core are assumed to move independently in an effective central potential

$$V_q(r) = (1+\gamma^0)V(r), \tag{2.1}$$

obeying the Dirac equation

$$\left[i\gamma^{\mu}\partial_{\mu} - m_{q} - V_{q}(r)\right]q(x) = 0, \tag{2.2}$$

and implying thereby a zeroth order Lagrangian density

$$\pounds_{q}^{0}(x) = \bar{q}(x)\left[\frac{i}{2}\gamma^{\mu}\overleftrightarrow{\partial}_{\mu} - m_{q} - V_{q}\right]q(x), \tag{2.3}$$

where $q(x)$ is the quark wave function and m_q is the quark mass.

Then under a global infinitesimal chiral transformation

$$q(x) \longrightarrow q(x) - i\gamma^{5}\left(\frac{\tau\cdot\varepsilon}{2}\right)q(x), \tag{2.4}$$

the axial vector current of the quarks

$$A_{q}^{\mu}(x) = \bar{q}(x)\gamma^{\mu}\gamma^{5}\frac{\tau}{2}q(x) \tag{2.5}$$

associated with such a transformation, is not conserved since its four divergence is

$$\partial_{\mu}A_{q}^{\mu}(x) = iG(r)\bar{q}(x)\gamma^{5}\tau q(x), \tag{2.6}$$

where $G(r) = m_q + V(r)$. This is due to the fact that the term $G(r)\bar{q}(x)q(x)$ in $\pounds_{q}^{0}(x)$ corresponding to quark mass m_q and the scalar potential $V(r)$ is chirally odd. The vector part of the potential poses no problem in this respect. Now in order to restore chiral symmetry in the usual manner, we can introduce a zero mass pion field with the interaction Lagrangian density

$$\pounds_{I}^{\pi}(x) = \frac{-i}{f_{\pi}}G(r)\bar{q}(x)\gamma^{5}(\tau,\phi)q(x), \tag{2.7}$$

where $f_\pi = 93$ MeV is the pion decay constant. Then the total axial vector current due to quark and pion together *i.e.*

$$A^{\mu}(x) = \bar{q}(x)\gamma^{\mu}\gamma^{5}\frac{\tau}{2}q(x) + f_{\pi}\partial^{\mu}\Phi(x) \tag{2.8}$$

gets conserved, with $\partial_{\mu}A^{\mu} = 0$. However, if we give the pion field a small but finite mass m_π, then

$$\partial_{\mu}A_{\mu}(x) = -f_{\pi}m_{\mu}^{2}\phi(x) \tag{2.9}$$

yields the usual PCAC relation.

First of all, leaving aside the quark-pion coupling, one can study the bare hadrons in terms of its individual quarks obeying the Dirac equation (2.2). Taking $V_q(r)$ as given in (2.1) with $V(r) = a \ln(r/b)$, $(a, b > 0)$, the Dirac equation (2.2) yields the spatial orbits for all the individual quarks in the low lying baryon ground states which can be written in their $1S_{1/2}$ configuration as

$$q(\boldsymbol{r}) = N_q \begin{bmatrix} g_q(\boldsymbol{r}) \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\lambda_q} g_q(\boldsymbol{r}) \end{bmatrix} \chi_\uparrow, \tag{2.10}$$

where $\lambda_q = E_q + m_q$,

$$g_q(\boldsymbol{r}) = A_q \frac{U_q(r)}{r} Y_0^0(\theta, \phi), \tag{2.11}$$

and N_q is the overall normalization constant given as

$$N_q = \left[\lambda_q / 2\left(E_q + a \ln b - a\langle \ln r\rangle\right)\right]^{1/2} \tag{2.12}$$

where $\langle \ln r\rangle$ is the expectation value of $\ln r$ with respect to $g_q(\boldsymbol{r})$.

With eqs. (2.10) and (2.11), the eq. (2.2) leads to the basic eigen value equation

$$U_q''(\rho) + \left(\epsilon_q - \ln(\rho)\right) U_q(\rho) = 0, \tag{2.13}$$

where $\rho = (r/r_0)$ is a dimensionless variable with

$$r_0 = \left(2a\lambda_q\right)^{-1/2}$$

and

$$\epsilon_q = \frac{1}{2}\left[\frac{E_q - m_q}{a} + \ln\left(2ab^2\lambda_q\right)\right]. \tag{2.14}$$

Eq. (2.13) can be solved for ϵ_q by a standard numerical method and one finds $\epsilon_q = 1.0443$ [3]. Once ϵ_q is known, eq. (2.14) can be solved to obtain the individual quark binding energy E_q. The solution $q(\boldsymbol{r})$ and E_q resulting from (2.2) can be utilized to describe the bare nucleons represented by the quark core alone. In fact, in an independent-quark model approach, where the quarks in a nucleon are assumed to satisfy the Dirac equation (2.2) with $V(r) = a\ln(r/b)$, a fairly reasonable description of the static properties of the bare nucleons has been obtained [5] in terms of magnetic moment μ_p, charge radius $\langle r^2\rangle_p^{1/2}$ and the axial vector coupling constant g_A for neutron beta decay. These properties have been estimated after the centre-of-mass correction as

$$\left(\mu_p, \langle r^2\rangle_p^{1/2}, g_A\right) = (2.662 \text{ nm}, 0.7663 \text{ fm}, 1.2485). \tag{2.15}$$

Here, the potential parameters $(a, b) = (0.1945$ GeV, 2.2914 GeV$^{-1})$, the quark masses $m_u = m_d = 0.1323$ GeV and as a consequence of eq. (2.14), the quark binding energy in the $1S_{1/2}$ configuration $E_u = E_d = 0.4915$ GeV have been used. Therefore in the present work, where our main objective is to build such a potential model for nucleons incorporating the chiral symmetry to study the pion coupling to quarks, we would adopt the same set of parameters that describes the bare nucleon properties in a reasonable manner.

3. Quark-pion coupling strength

We would like to study mainly the coupling of quarks in a nucleon to pions in a chiral symmetric potential model. It is a fact that chiral SU(2) × SU(2) is experimentally found to be an excellent symmetry of strong interaction having its physical realization in pion with its small mass as the corresponding Goldstone boson. Therefore, we concentrate our discussion mainly in the (u, d) flavour sector. As a first step in this direction, let us assume that the interaction Lagrangian density in (2.7) can be written effectively as

$$\mathcal{L}_I^\pi = -iG_{qq\pi}\,\bar{q}(x)\gamma^5(\tau\cdot\phi)q(x), \tag{3.1}$$

with $G_{qq\pi}$ as the effective quark-pion coupling strength. Then in a classical field approximation, taking the emitted pion field ϕ_i in the process $q \to q + \pi$ as a plane wave with momentum $\boldsymbol{k}$, we can write the interaction Hamiltonian as

$$H_{\rm int} \simeq iG_{qq\pi}\int d^3r\,\bar{q}(\boldsymbol{r})\gamma^5 q(\boldsymbol{r})\exp(i\boldsymbol{k}\cdot\boldsymbol{r})\,\tau_j. \tag{3.2}$$

Now from (2.7) we can similarly obtain

$$H_{\rm int} \simeq \frac{1}{f_\pi}\int d^3\,\bar{q}(\boldsymbol{r})\gamma^5 q(\boldsymbol{r})G(r)\exp(i\boldsymbol{k}\cdot\boldsymbol{r})\,\tau_j. \tag{3.3}$$

Then comparing (3.2) and (3.3), we can obtain a much simpler

$$G_{qq\pi} = \frac{1}{f_\pi}\,\frac{\int d^3r\,G(r)\bar{q}(\boldsymbol{r})\gamma^5 q(\boldsymbol{r})\exp(i\boldsymbol{k}\cdot\boldsymbol{r})}{\int d^3r\,\bar{q}(\boldsymbol{r})\gamma^5 q(\boldsymbol{r})\exp(i\boldsymbol{k}\cdot\boldsymbol{r})}. \tag{3.4}$$

Then taking the $1S_{1/2}$ spatial wave functions of the quarks in (2.10), we obtain

$$G_{qq\pi} = \frac{1}{f_\pi}\Big[(m_q - a\ln b) + a\big(\langle \ln r\, j_0(kr)\rangle \langle j_1(kr)/kr\rangle\big)\big/\langle j_0(kr)\rangle\Big], \tag{3.5}$$

where $J_0(kr)$ and $J_1(kr)$ are the spherical Bessel function of order zero and one respectively. The angular brackets appearing in (3.5) are the expectation values with respect to $g_q(r)$.

Then with a soft pion approximation, we can approximate

$$G_{qq\pi} \simeq \frac{1}{3f_\pi}\Big[a + 3(m_q - a\ln b) + 3a\langle\ln r\rangle\Big] \tag{3.6}$$

The expectation value $<\ln r>$ with respect to $g_q(r)$ can be easily obtained as [3]

$$\langle \ln r \rangle = (E_q - m_q - a + 2a \ln b)/2a \tag{3.7}$$

with which the eq. (3.6) can be expressed as

$$G_{qq\pi} \simeq (3E_q + 3m_q - a)/6f_\pi \tag{3.8}$$

so that, the quark-pion coupling constant comes out as

$$G^2_{qq\pi}/4\pi = 0.719. \tag{3.9}$$

This is comparable with the estimate obtained by Suzuki and Bhaduri [6] from the ratio

$$\Gamma(\rho \longrightarrow \pi^- \pi^0)/\Gamma(\rho \longrightarrow \pi^- \gamma).$$

A better estimate of the quark-pion coupling constant can be obtained in a more reasonable way by looking at the NNπ-vertex. For the interaction Lagrangian density (2.7), the NNπ-vertex function, in a point pion approximation can be written as

$$V_j^{N'N}(k) = \frac{-i}{f_\pi}(2\omega_k)^{-1/2} \int d^3 r G(r) \exp(i\boldsymbol{k}\cdot\boldsymbol{r}) \langle N'| \sum_q \bar{q}(r)\gamma^5 q(r)\tau_j |N\rangle. \tag{3.10}$$

Here, j is the isospin index and $\omega_k = (k^2 + m_\pi^2)^{1/2}$ is the pion energy. Since for the NNπ-vertex, the spatial orbits of the quarks in the initial and final nucleon states are the same as $1S_{1/2}$ ones, using eq. (2.10) in eq. (3.10), we can obtain

$$V_j^{N'N}(k) = \frac{-i}{f_\pi}(2\omega_k)^{-1/2} \frac{N_q^2}{\lambda_q} I(k) \langle N'| \sum_q (\sigma_q \cdot k)\tau_j |N\rangle, \tag{3.11}$$

where
$$I(k) = \frac{8\pi}{k}\int_0^\infty dr r^2 G(r) j_1(kr) g_q(r) g'_q(r). \tag{3.12}$$

Now with evaluation of $I(k)$, the eq. (3.11) can be obtained as

$$\begin{aligned} V_j^{N'N}(k) &= \frac{i}{2f_\pi}(2\omega_k)^{-1/2}\left[\frac{3}{5} g_A u(k)\right] \langle N'| \sum_q (\sigma_q \cdot k)\tau_j |N\rangle \\ &= \langle N'| \sum_q V_j^{qq}(k) |N\rangle, \end{aligned} \tag{3.13}$$

where g_A is the axial vector coupling constant which can be obtained in this model as

$$g_A = \frac{5}{9}\left(4N_q^2 - 1\right) \tag{3.14}$$

and $u(k)$ is the vertex form factor given by [5]

$$u(k) = \left(\frac{10N_q^2}{3\lambda_q g_A}\right)\Big[(m_q - a\ln b)\,\langle j_0(kr)\rangle$$

$$+ a\langle \ln r j_0(kr)\rangle + a\langle j_1(kr)/kr\rangle\Big] \tag{3.15}$$

which for $k \to 0$ reduces to one as expected. Here N_q^2 given by eq. (2.12) can be expressed in a simplified form as [3]

$$N_q^2 = \lambda_q/(a+\lambda_q). \tag{3.16}$$

Now the eq. (3.13) yields the quark-pion vertex operator function as

$$V_i^{qq}(k) = \frac{i}{2f_\pi}(2\omega_k)^{-1/2}\left[\frac{3}{5}g_A\, u(k)\right](\sigma_q\cdot k)\tau_i. \tag{3.17}$$

The corresponding expression in Chew-Low type model [7] is written in terms of the pseudo-vector qqπ-coupling $f_{qq\pi}$ as

$$V_i^{qq}(k) = i(2\omega_k)^{-1/2}\sqrt{4\pi}\left(f_{qq\pi}/m_\pi\right)u(k)(\sigma_q\cdot k)\tau_i. \tag{3.18}$$

Comparing eqs. (3.17) and (3.18) we have

$$\sqrt{4\pi}\left(f_{qq\pi}/m_\pi\right) = \frac{1}{2f_\pi}\left(3g_A/5\right). \tag{3.19}$$

This is the equivalent Goldberger-Treiman relation, which with the familiar equivalence of pseudo-scalar and pseudo-vector coupling strength, yields

$$\left(G_{qq\pi}/2\overline{m}_q\right) = \sqrt{4\pi}\left(f_{qq\pi}/m_\pi\right) = \frac{1}{2f_\pi}\left(3g_A/5\right), \tag{3.20}$$

where $\overline{m}_q$ is the effective constituent quark mass taken as one third of the N–Δ spin-isospin average mass *i.e.* 391 MeV. Then, we have

$$\left(G_{qq\pi}^2/4\pi\right) = \frac{1}{4\pi}\left(\frac{\overline{m}_q^2}{f_\pi}\right)^2\left[\frac{3}{5}g_A\right]^2 \simeq 0.656. \tag{3.21}$$

However, if we consider the center of mass correction for g_A then using the corrected g_A value from (2.15) we get

$$\left(G_{qq\pi}^2/4\pi\right)_0 \simeq 0.789. \tag{3.22}$$

The pions coupling to the quark have been considered so far to be point particles. But one can introduce the finite size of the pion according to the prescription of Refs. [7]

and [8] by visualizing the pion absorption as a process in which quark of the bare nucleon is replaced by a quark of the pion after it is annihilated by the antiquark of the pion. Due to finite size of the pion, the NNπ-vertex function gets modified and can be written as

$$V_j^{N'N}(k) = \frac{-i}{f_\pi}(2\omega_k)^{-1/2}\int d^3r\, d^3\rho\, G(r)\exp(ik\cdot r)$$

$$\times P(\rho)\langle N'|\sum_q \bar{q}(r+\rho/2)\gamma^5\tau_j q(r-\rho/2)|N\rangle, \tag{3.23}$$

where ρ is the $q\bar{q}$-separation distance and $P(\rho)$ is the probability function for finding such $q\bar{q}$ pair in the pion. Now introducing a size parameter R_π for the pion, one can choose

$$P(\rho) = \frac{3}{4\pi R_\pi^2}\theta(R_\pi - \rho). \tag{3.24}$$

However, with the reasonable assumption that

$$q(r+\rho/2) = q(r)\exp\left[-(\rho/2)^2/2r_0^2\right] \tag{3.25}$$

where the ρ-dependent part of the quark wave function is taken in Gaussian form, the eq. (3.23) can be simplified to give the quark-pion vertex operator function.

$$V_j^{qq}(k) \simeq \frac{i}{2f_\pi}(2\omega_k)^{-1/2}\frac{3g_A}{5}u(k)F(R_\pi)(\sigma_q.k)\tau_j, \tag{3.26}$$

where $$F(R_\pi) = \int d^3\rho\, p(\rho)\exp\left(-\rho^2/4r_0^2\right) \tag{3.27}$$

The integral (3.27) can be simplified to give $F(R_\pi)$ as

$$F(R_\pi) = \frac{12r_0^3}{R_\pi^3}\gamma\left(3/2, R_\pi^2/4r_0^2\right)$$

$$\simeq \left[1+\frac{1}{10}(R_\pi/r_0)^2\right]\exp\left(-R_\pi^2/4r_0^2\right). \tag{3.28}$$

Then proceeding as before and taking the centre of mass correction for g_A into account in (3.26), we can obtain

$$\left(G_{qq\pi}^2/4\pi\right) = \left(G_{qq\pi}^2/4\pi\right)_0 F^2(R_\pi), \tag{3.29}$$

where $(G_{qq\pi}^2/4\pi)_0$ is the coupling strength obtained with point pion approximation [eq. 3.22)]. It is obvious that the effect of the finite size is to reduce the coupling strength depending upon the size parameter R_π. Here R_π is expected not to be the pion charge radius but rather the radius of the $q\bar{q}$-pair distribution within the pion, which is observed to be considerably smaller [9] than the charge radius of the pion. According to the estimate of Sujuki and Bhaduri [10], $R_\pi = 0.4$ fm or smaller. Similar values are also obtained in a microscopic chiral model of the pion [11]. Therefore, taking a range of values for R_π as 0.4, 0.3 and 0.2 fm respectively, we obtain

$$\left(G^2_{qq\pi}/4\pi\right) = (0.58, 0.67, 0.73). \tag{3.30}$$

The quark pion coupling strength can however, be estimated with a different choice of the probability function $P(\rho)$. One can choose this function in the following Gaussian form :

$$P(\rho) = \frac{1}{(2\pi)^{3/2} R^3_\pi} \exp\left(-\rho^2/2R^2_\pi\right). \tag{3.31}$$

Then writing the vertex function (3.23) in the form

$$V_j^{N'N}(k) = \frac{-1}{f_\pi}(2\omega_k)^{-1/2} \int d^3r d^3\rho G(r) \exp[i(r+\rho).k]$$

$$\times P(\rho) \langle N' \left| \sum_q \bar{q}(r)\gamma^5 \tau_j q(r) \right| N \rangle \tag{3.32}$$

and using (3.31) for $P(\rho)$, one can get the quark-pion vertex function operator $V_j^{qq}(k)$ in the form :

$$V_j^{qq}(k) = \frac{1}{2f_\pi}(2\omega_k)^{-1/2} \frac{3g_A}{5} u(k) \exp\left(-k^2 R^2_\pi/2\right)(\sigma_q .k)\tau_j. \tag{3.33}$$

Thus, the vertex form factor $u(k)$ now modifies due to an additional momentum dependent multilating factor $\exp\left(-k^2 R^2_\pi/2\right)$.

In fact, the quark-pion coupling is defined at $k^2 = -m^2_\pi$ which when taken into account, yields

$$\left(G^2_{qq\pi}/4\pi\right) = \left(G^2_{qq\pi}/4\pi\right)_0 u^2\left(k^2 = -m^2_\pi\right)\exp\left(m^2_\pi R^2_\pi/2\right). \tag{3.34}$$

For a range of value for R_π as 0.4, 0.3 and 0.2 fm respectively, we obtain

$$\left(G^2_{qq\pi}/4\pi\right) = (0.71, 0.698, 0.69). \tag{3.35}$$

4. Nucleon-pion coupling constant

In the foregoing Section, we have provided estimates of the pion-quark coupling in the present model with the approximation of a point pion as well as finite size pion.

However, a more direct effect of pion coupling is observable normally in terms of pion-nucleon. Therefore, we would like to obtain here the pion-nucleon coupling constant in the present model.

The pion-nucleon coupling can be accounted for by considering the term $\mathcal{L}_\pi = \mathcal{L}_\pi^0 + \mathcal{L}_I^\pi$ in the Lagrangian density, where $\mathcal{L}_\pi^0 = \frac{1}{2}[(\partial_\mu \phi)^2 - m_\pi^2 (\phi)^2]$ and $\mathcal{L}_I^\pi$ is provided through eq. (3.1). The pion field $\phi^\lambda(x)$ satisfies the equation

$$(\Box + m_\pi^2)\phi^\lambda(x) = J_5^\lambda(x), \tag{4.1}$$

where the source function J_5^λ provides the coupling of a pion to the quarks in the nucleon core and is given by

$$J_5^\lambda(x) = \frac{-1}{f_\pi} G(r) \sum_q \bar{q}(x)\gamma^5 \tau^\lambda q(x). \tag{4.2}$$

The pion-nucleon form factor $G_{NN\pi}(k^2)$ is defined as

$$iG_{NN\pi}(k^2)\langle \sigma_N . k\tau_N^\lambda \rangle = 2M_N \left\langle N \middle| \int d^3 r \exp(ik.r) J_5^\lambda(r) \middle| N \right\rangle \tag{4.3}$$

where σ_N and τ_N^λ refer to nucleon spin and isospin operators to be taken between nucleon states. If one does not take the recoil effect into account, then for the three quarks in $1S_{1/2}$ orbits as given in (2.10), one easily gets

$$G_{NN\pi}(k^2) = (M_N / f_\pi) g_A u(k), \tag{4.4}$$

where $u(k)$ is given by eq. (3.15). Defining the pseudo-scalar pion-nucleon coupling constant as $g_{NN\pi} = G_{NN\pi}(k^2 = -m_\pi^2)$, one obtains from (4.4)

$$g_{NN\pi} = (M_P / f_\pi) g_A u(k^2 = -m_\pi^2). \tag{4.5}$$

Substituting the value of g_A corrected for the centre-of-mass motion in eq. (4.5), one finds

$$(g_{NN\pi}^2 / 4\pi) = 14.539, \tag{4.6}$$

which agrees reasonably well with the corresponding experimental value 14.1. Also the bare pseudo-vector $NN\pi$-coupling constant $f_{NN\pi}$ can be computed from the usual expression

$$\sqrt{4\pi} \frac{f_{NN\pi}}{m_\pi} = \frac{g_{NN\pi}}{2M_P} \tag{4.7}$$

and one obtains $f_{NN\pi} = 0.2844$ which is in excellent agreement with its observed value 0.283.

The finite size of the pion, however, modifies (4.6) as

$$(g_{NN\pi}^2 / 4\pi) = (g_{NN\pi}^2 / 4\pi)_0 F^2(R_\pi), \tag{4.8}$$

where $F(R_\pi)$ is given by (3.28). It is clear that the finite size of the pion reduces the value of $(g_{NN\pi}^2 / 4\pi)_0$ given in (4.6) to 10.68, 12.25 and 13.48 for range of values for R_π as 0.4, 0.3 and 0.2 fm respectively.

5. Conclusion

The value of the coupling strength $(G^2_{qq\pi}/4\pi)$ determined by Suzuki and Bhaduri [10] from the vector meson decay width ratios with a static approximation, can be cited here for a comparison. They obtained it as about (i) 0.4 from $\left((\bar{\rho} \to \bar{\pi}\gamma)/\Gamma(\bar{\rho} \to e^+e^-)\right)$, (ii) 0.5 from $\Gamma(\omega \to \pi^0\gamma)/\Gamma(\omega \to e^+e^-)$ and (iii) 0.88 from $\Gamma(\rho \to \bar{\pi}\pi^0)/(\rho \to \bar{\pi}\gamma)$. We find that the value of the quark-pion coupling constant extracted from the experimental vector meson decay widths for case (iii) is quite comparable with our theoretical estimates in this model given in eqs. (3.9), (3.21), (3.22), (3.30) and (3.35). However, from the observations of [5] examining the decay of excited N and Δ states, one obtains $G^2_{qq\pi}/4\pi \simeq 1.1$ which is larger than our estimate.

The nucleon-pion coupling constant $(g^2_{NN\pi}/4\pi)_0$ in this model, comes out to be 14.539 which is in excellent agreement with the experimental value 14.1. The finite size of the pion, however, reduces the value of $(g^2_{NN\pi}/4\pi)$ to 10.68, 12.25 and 13.48 for R_π taken as 0.4, 0.3 and 0.2 fm respectively. The value for the pseudo vector NNπ-coupling constant $f_{NN\pi}$ in the present model comes out to be 0.2844 as against the experimental value 0.283.

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